Journal of Engineering Physics and Thermophysics, Vol. 79, No. 1, 2006

CONTACT INTERACTION BETWEEN A RIGID DIE AND AN ELASTIC LAYER IN NONSTATIONARY FRICTIONAL HEAT GENERATION

P. P. Krasnyuk UDC 539.3

Consideration is given to a new quasistatic contact thermoelasticity problem for an elastic layer fixed at its base and on whose surface a rigid heat-insulated die moves, when the interaction of the bodies is accompanied by the heat generation from the action of friction forces. Under the assumption that the process of heat generation is nonstationary, the problem has been reduced to an integral equation with integration limits varying with time. It has been shown that an increase in the heat-generation intensity produces a decrease in the upsetting of the die and equilibrium is possible for its negative values with decrease in the contact portion; for a die with a plane base this is equivalent to the separation of the body from the layer at the edges of the interaction interval.

Quasistationary heat generation on the portion of contact of two elastic bodies in a plane formulation was considered for the first time in [1, 2], and the motion of a die over the surface of an elastic half-plane in quasistationary heat release was considered in [3, 4]. In the last work, the critical value of the velocity of motion in thermal explosion or under a sharp change (bifurcation) in the contact temperatures was found.

Plane contact problems in nonstationary heat generation caused by the motion of a die over the surface of an elastic half-plane were investigated in [5, 6]. It has been shown that a monotone variation in the contact portion with time is one principal effect accompanying the nonstationary heat generation in contact with friction. If the stationary value of the half-width of the contact portion is close to the critical value, the region of contact, upon reaching this value, will be multiply connected with contact zones and a detachment [7].

Thermoelastic contact problems with allowance for nonstationary frictional heat generation were investigated earlier either for bodies modeled by an elastic half-space or for a thick layer, when the first term of the asymptotics of the regular part of the kernels of integral equations was selected ("method of large λ" [8, 9]). Below, we construct the exact solution for a layer whose contact surface beyond the interaction region is heat-insulated, which enables us to reduce the problem formulated to one integral equation with integration limits varying with time.

Formulation of the Problem. Let a rigid bar die be pressed by the force $P(\tau)$ referred to a unit length and applied with an eccentricity ε to an elastic layer of thickness h, which is rigidly fixed at the base (Fig. 1). The region of the initial contact Ω_0 between the die and the layer is described by the inequalities $-b \le x \le a$ and $|z| < \infty$, whereas the shape of the body's base in the contact region is determined by the function $y = f_0(x)$. We assume that the die moves over the layer surface with a low velocity $v(\tau)$ in the direction of the z axis. Due to the action of the friction forces τ_{yz} which occur on the contacting surfaces and obey the Amonton law ($\tau_{yz} = f\sigma_y$), we have heat generation in the contact plane; as a consequence of the heat insulation of the die, the entire heat generated on the contact is directed into the layer, causing its heating, which is responsible for the swelling of the body's contacting surface and consequently for the time variation in the boundary of the portion of interaction near the die's edge. Heat exchange following the Newton law occurs between the lower plane of the layer and the ambient medium, whose temperature is taken to be zero; the upper plane beyond the interaction portion is assumed to be heat-insulated.

Under the above assumptions realizing plane deformation in the layer, the problem is reduced to construction of the solutions of the system involving the differential equations of heat conduction

$$\Delta T = k^{-1}\partial_\tau T \tag{1}$$

L'vov State Financial Academy, 3 Kopernik Str., L'vov, 79000, Ukraine. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 79, No. 1, pp. 185–195, January–February, 2006. Original article submitted August 13, 2004.

1062-0125/06/7901-0191

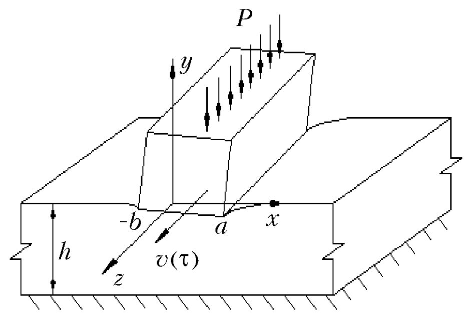


Fig. 1. Scheme of the problem of contact interaction between a rigid die and an elastic layer.

and thermal elasticity

$$(1-2\nu)\,\Delta u_x + \partial_x\,(\partial_x u_x + \partial_y u_y) = 2\alpha\,(1+\nu)\,\partial_x T\,,$$

$$(1-2\nu)\,\Delta u_y + \partial_y\,(\partial_x u_x + \partial_y u_y) = 2\alpha\,(1+\nu)\,\partial_y T\,, \tag{2}$$

which satisfy the initial

$$T\,(x, y, 0) = 0\,, \quad c\,(0) = b\,, \quad d\,(0) = a\,, \tag{3}$$

boundary and contact conditions

$$y = -\,h: \quad 1)\ \partial_y T = \gamma T\,, \quad 2)\ u_x = 0\,, \quad 3)\ u_y = 0\,, \tag{4}$$

$$y = 0: \quad -\,c\,(\tau) \le x \le d\,(\tau):$$

$$1)\ \partial_y T = fv\,(\tau)\,\lambda^{-1} p\,(x, \tau)\,, \quad 2)\ u_y = -\,\delta_0\,(\tau) - \alpha_0\,(\tau)\,x + f_0\,(x)\,, \quad 3)\ \tau_{yx} = 0\,; \tag{5}$$

$$x < -\,c\,(\tau)\,, \quad x > d\,(\tau): \quad \partial_y T = 0\,, \quad \sigma_y = 0\,, \quad \tau_{yx} = 0\,. \tag{6}$$

The stresses are determined from the formulas

$$\sigma_x = \frac{E\,(1-\nu)}{(1+\nu)\,(1-2\nu)}\left(\partial_x u_x + \frac{\nu}{1-\nu}\,\partial_y u_y - \alpha\,\frac{1+\nu}{1-\nu}\,T\right),$$

$$\sigma_y = \frac{E\,(1-\nu)}{(1+\nu)\,(1-2\nu)}\left(\partial_y u_y + \frac{\nu}{1-\nu}\,\partial_x u_x - \alpha\,\frac{1+\nu}{1-\nu}\,T\right), \quad \tau_{xy} = \frac{E}{2\,(1+\nu)}\,(\partial_y u_x + \partial_x u_y)\,. \tag{7}$$

Furthermore, the equilibrium conditions

$$\int\limits_{-c(\tau)}^{d(\tau)} p\,(x, \tau)\,dx = P\,(\tau)\,, \quad \int\limits_{-c(\tau)}^{d(\tau)} xp\,(x, \tau)\,dx = \varepsilon P\,(\tau) \tag{8}$$

and, upon reaching the steady-state regime, the condition of thermal balance must hold

$$\lim_{\tau\to\infty}\int\limits_{-\infty}^{\infty}\partial_y T\,(x, 0, \tau)\,dx = \gamma\lim_{\tau\to\infty}\int\limits_{-\infty}^{\infty} T\,(x, -\,h, \tau)\,dx\,.$$

Solution of the Problem. The use of the integral transformations of Fourier in the coordinate x and Laplace in τ [10] in solving thermoelasticity problem (1)–(7) enables us to obtain the integral transforms for the temperature, displacements, and stresses of the layer in terms of the unknown function of the contact pressure $p(x, \tau)$; this function is determined by solution of the integral equation which is given here in dimensionless form. Referring the linear dimensions of the body to the layer thickness h, the stresses to the quantity P_0/h, and the temperature to $\alpha Eh/(2P_0(1-\nu))$, consequently we have

$$\frac{1}{\pi} c_0(\mathrm{Fo}) \int_{-1}^{1} p(t, \mathrm{Fo})\, \Delta(c_0(\mathrm{Fo})(t-x))\, dt - \frac{\chi}{\pi} \partial_{\mathrm{Fo}} \int_0^{\mathrm{Fo}} c_0(\eta)\, v_*(\eta) \int_{-1}^{1} p(t, \eta)\, H(c_0(\eta)\, t +$$

$$+ c_+(\eta) - c_0(\mathrm{Fo})\, x - c_+(\mathrm{Fo}),\ \mathrm{Fo} - \eta)\, dt d\eta =$$

$$= \delta_*(\mathrm{Fo}) + \alpha_*(\mathrm{Fo})(c_0(\mathrm{Fo})\, x + c_+(\mathrm{Fo})) - f_*(c_0(\mathrm{Fo})\, x + c_+(\mathrm{Fo})),\ (|x| \le 1). \tag{9}$$

This equation together with the conditions of equilibrium of the die

$$c_0(\mathrm{Fo}) \int_{-1}^{1} p(x, \mathrm{Fo})\, dx = P_*(\mathrm{Fo}), \quad c_0^2(\mathrm{Fo}) \int_{-1}^{1} x p(x, \mathrm{Fo})\, dx = P_*(\mathrm{Fo})(\varepsilon - c_+(\mathrm{Fo})) \tag{10}$$

and the conditions of boundedness of contact stresses (unknown contact region)

$$p(\pm 1) = 0 \tag{11}$$

yield the complete system of equations of the problem formulated. For the layer temperature, we obtain the integral transform

$$T(x, y, \mathrm{Fo}) = \frac{\chi}{\pi} \partial_{\mathrm{Fo}} \int_0^{\mathrm{Fo}} c_0(\eta)\, v_*(\eta) \int_{-1}^{1} p(t, \eta)\, \Phi(c_0(\eta)\, t + c_+(\eta) - c_0(\mathrm{Fo})\, x - c_+(\mathrm{Fo}), y, \mathrm{Fo} - \eta)\, dt d\eta.$$

Here we have

$$\Delta(x) = \int_0^{\infty} \overline{\Delta}(\xi) \cos(\xi x)\, d\xi; \quad \overline{\Delta}(\xi) = \xi^{-1} \frac{(3-4\nu)\cosh(\xi)\sinh(\xi) - \xi}{\xi^2 - (1-2\nu)^2 \sinh^2(\xi) + 4(1-\nu)^2 \cosh^2(\xi)};$$

$$\Phi(x, y, \mathrm{Fo}) = \int_0^{\infty} \overline{\Phi}_{\mathrm{st}}(\xi, y) \cos(\xi x)\, d\xi - \frac{\pi}{2} \sum_{m=1}^{\infty} \frac{\cos(\mu_m y)}{\mu_m (1 + \mathrm{Bi}(\mu_m^2 + \mathrm{Bi}^2)^{-1})} \times$$

$$\times \sum_{k=1}^{2} \exp((-1)^k \mu_m x)\, \mathrm{erfc}\left(\mu_m \sqrt{\mathrm{Fo}} + (-1)^k \frac{x}{2\sqrt{\mathrm{Fo}}}\right);$$

$$\overline{\Phi}_{\mathrm{st}}(\xi, y) = \xi^{-1} \frac{\xi \cosh(\xi(1+y)) + \mathrm{Bi}\sinh(\xi(1+y))}{\xi \sinh(\xi) + \mathrm{Bi}\cosh(\xi)}, \quad H(x, \mathrm{Fo}) = \int_0^{\infty} \overline{H}(\xi, \mathrm{Fo}) \cos(\xi x)\, d\xi;$$

$$\overline{H}(\xi, \mathrm{Fo}) = \overline{H}_{\mathrm{st}}(\xi) + 4 \frac{2(1-\nu)(\xi \sinh(\xi) - \mathrm{Bi}\cosh(\xi)) - \xi(\sinh(\xi) + \xi\cosh(\xi) + \mathrm{Bi}\sinh(\xi))}{\xi^2 - (1-2\nu)^2 \sinh^2(\xi) + 4(1-\nu)^2 \cosh^2(\xi)} \times$$

$$\times \sum_{m=1}^{\infty} \frac{1}{(\mu_m^2 + \xi^2)^2} \frac{\cos(\mu_m) \exp(-(\xi^2 + \mu_m^2)\,\text{Fo})}{1 + \text{Bi}\,(\mu_m^2 + \text{Bi}^2)^{-1}} + 4\, \frac{\xi^2 - (3 - 4\nu)\,\xi \sinh(\xi) \cosh(\xi)}{\xi^2 - (1 - 2\nu)^2 \sinh^2(\xi) + 4\,(1 - \nu)^2 \cosh^2(\xi)} \times$$

$$\times \sum_{m=1}^{\infty} \frac{1}{(\mu_m^2 + \xi^2)^2} \frac{\exp(-(\xi^2 + \mu_m^2)\,\text{Fo})}{1 + \text{Bi}\,(\mu_m^2 + \text{Bi}^2)^{-1}} \,;$$

$$\overline{H}_{\text{st}}(\xi) = \frac{1}{\xi^2} \frac{(\xi^2 + (3 - 4\nu)\sinh^2(\xi))\,(\xi \sinh(\xi) + \text{Bi}\cosh(\xi)) + 2\,(1 - \nu)\,\xi\,(\xi \cosh(\xi) - \text{Bi}\sinh(\xi) + \sinh(\xi))}{(\xi \cosh(\xi) + \text{Bi}\cosh(\xi))\,(\xi^2 - (1 - 2\nu)^2 \sinh^2(\xi) + 4\,(1 - \nu)^2 \cosh^2(\xi))} \,;$$

$$\chi = \frac{\alpha E h f v_0}{2\lambda\,(1 - \nu)} \,; \quad \delta_* = \frac{\delta_0 E}{2 P_0\,(1 - \nu^2)} \,; \quad \alpha_* = \frac{\alpha_0 E h}{2 P_0\,(1 - \nu^2)} \,;$$

$$f_* = \frac{f_0 E}{2 P_0\,(1 - \nu^2)} \,; \quad c_0(\text{Fo}) = (d(\text{Fo}) + c(\text{Fo}))/(2h) \,;$$

$$c_+(\text{Fo}) = (d(\text{Fo}) - c(\text{Fo}))/(2h) \,; \quad v(\tau) = v_0 v_*(\text{Fo}) \,;$$

$$\mu_m \sin(\mu_m) - \text{Bi}\cos(\mu_m) = 0 \,; \quad -1 \le y \le 0 \,.$$

Into the formulas given above, we have not introduced new variables for the coordinates x, y and eccentricity ε referred to the layer thickness h and for the contact-pressure function $p(x, \text{Fo})$ referred to the combination of the parameters P_0/h. The unknown boundaries of the contact region $c_0(\text{Fo})$ and $c_+(\text{Fo})$ are obtained from the condition of boundedness of contact stresses $p(\pm 1, \text{Fo}) = 0$; this condition is used only for such a state of the tribosystem when a change in the p sign is observed at the ends of the interval $x \in [-1, 1]$. Otherwise, we will have $c_0(\text{Fo}) = (a + b)/(2h)$ and $c_+(\text{Fo}) = (a - b)/(2h)$.

Based on the trapezium method [11], we carry out time discretization of the integral equation (9) with conditions (10) and (11) in the time interval $[0, \text{Fo}_*]$ in which the behavior of the tribosystem is investigated (this interval is subdivided into N time intervals $\text{Fo}_k = k\text{Fo}_1$ ($k = 0, ..., N$), where $\text{Fo}_N = \text{Fo}_*$). Then, at each instant of time Fo_k, we obtain the integral equation

$$\frac{1}{\pi} c_0(\text{Fo}_k) \int_{-1}^{1} p(t, \text{Fo}_k)\,(\Delta(c_0(\text{Fo}_k)(t - x)) - 0.5\chi v_*(\text{Fo}_k)\,H(c_0(\text{Fo}_k)(t - x), \text{Fo}_1))\,dt =$$

$$= \delta_*(\text{Fo}_k) + \alpha_*(\text{Fo}_k)\,(c_0(\text{Fo}_k)\,x + c_+(\text{Fo}_k)) - f_*(c_0(\text{Fo}_k)\,x + c_+(\text{Fo}_k)) + \frac{\chi}{\pi} R'(x, \text{Fo}_k) \,, \quad |x| \le 1 \tag{12}$$

with the conditions

$$p(\pm 1) = 0 \,, \quad c_0(\text{Fo}_k) \int_{-1}^{1} p(x, \text{Fo}_k)\,dx = P_*(\text{Fo}_k) \,, \quad c_0^2(\text{Fo}_k) \int_{-1}^{1} x p(x, \text{Fo}_k)\,dx = P_*(\text{Fo}_k)\,(\varepsilon - c_*(\text{Fo}_k)) \,,$$

where

$$R'(x, 0) = 0 \,; \quad R'(x, \text{Fo}_1) = 0.25 G_1'(x, \text{Fo}_{0,2}) \,;$$

$$R'(x, \text{Fo}_2) = 0.5 G_2'(x, \text{Fo}_{1,2}) + 0.25\,(G_2'(x, \text{Fo}_{0,3}) - G_2'(x, \text{Fo}_{0,1})) \,;$$

$$R'(x, \mathrm{Fo}_n) = 0.5 G'_n(x, \mathrm{Fo}_{n-1,2}) + 0.5 \sum_{k=1}^{n-2} (G'_n(x, \mathrm{Fo}_{k,n+1-k}) - G'_n(x, \mathrm{Fo}_{k,n-1-k})) +$$

$$+ 0.25\,(G'_n(x, \mathrm{Fo}_{0,n+1}) - G'_n(x, \mathrm{Fo}_{0,n-1}))\,, \quad n \geq 3\,;$$

$$G'_m(x, \mathrm{Fo}_{i,j}) = v_*(\mathrm{Fo}_i)\, c_0(\mathrm{Fo}_i) \int_{-1}^{1} p(t, \mathrm{Fo}_i)\, H(c_0(\mathrm{Fo}_i)\, t + c_+(\mathrm{Fo}_i) - c_0(\mathrm{Fo}_m)\, x - c_+(\mathrm{Fo}_m), \mathrm{Fo}_j)\, dt\,.$$

In solving (12), we will use the Multhopp–Kalandiya methods [12] and the theorems given in [8, 9].

As a result of any asymptotic analysis of the kernels of the integral equations with allowance for the relation from [8, 9], we may state that the kernels $H(x, \mathrm{Fo})$ and $\Phi(x, y, \mathrm{Fo})$ will be regular for $y \neq 0$ and $\mathrm{Fo} > 0$, whereas the kernels $\Delta(x)$ and $\Phi(x, 0, \mathrm{Fo})$ $(\mathrm{Fo} > 0)$ have a logarithmic singularity. Then we represent the contact pressure as

$$p(x, \mathrm{Fo}_k) = \frac{\psi(x, \mathrm{Fo}_k)}{\sqrt{1 - x^2}}\,, \tag{13}$$

where $\psi(x, \mathrm{Fo}_k)$ is the continuously differentiable and bounded function for which we select representation in the form of the interpolation Lagrange polynomial of the nth degree [13] in the Chebyshev polynomials of the first kind $T_m(x)$:

$$\psi(x, \mathrm{Fo}_k) = \frac{1}{n} \sum_{i=1}^{n} \psi(x_i, \mathrm{Fo}_k) \left(1 + 2 \sum_{m=1}^{n-1} T_m(x_i)\, T_m(x) \right), \tag{14}$$

where $x_i = \cos\left(\frac{2i-1}{2n}\pi\right)$ $(i = 1, \ldots, n)$ are the zeros of the Chebyshev polynomial of the first kind of order n [14].

Substituting the expression for contact pressure (13) into the integral equation (12), we accurately compute the integrals with logarithms from the known formulas and approximately find the values of the regular integrals from the Gauss formulas [11]. Then we reduce Eq. (12) at each instant of time Fo_k to a system of linear equations for the expansion coefficients in the interpolation polynomial that totally determine the change in the contact pressure at this instant of time.

Prescribing the boundaries of contact $c_0(\mathrm{Fo}_k)$ and $c_+(\mathrm{Fo}_k)$ and the value of the eccentricity ε, we select such values of the parameters $\delta_*(\mathrm{Fo}_k)$ and $\alpha_*(\mathrm{Fo}_k)$ that the contact pressure satisfies the integral equilibrium relations. The conditions $\psi(-1, \mathrm{Fo}_k) > 0$ and $\psi(1, \mathrm{Fo}_k) > 0$ must hold. Violation of one condition for $+1$ or -1 is equivalent to the separation of the die base at this edge. Then, selecting the boundaries of the contact region, we strive for the fulfillment of the approximate relations following from the numerical approach to solution of the system $|\psi(-1, \mathrm{Fo}_k)| < \varepsilon_0$ or $|\psi(1, \mathrm{Fo}_k)| < \varepsilon_0$, where ε_0 is a certain number determining the computational error ($\varepsilon_0 \approx 10^{-5}$, as a rule). Based on [9], the fulfillment of the last two conditions is equivalent to the fact that

$$\begin{gathered} p(x, \mathrm{Fo}_k) = \psi_1(x, \mathrm{Fo}_k) \sqrt{\frac{1+x}{1-x}}\,, \quad \text{if} \quad \psi(-1, \mathrm{Fo}_k) = 0 \quad \text{or} \\ p(x, \mathrm{Fo}_k) = \psi_2(x, \mathrm{Fo}_k) \sqrt{1 - x^2}\,, \quad \text{if} \quad \psi(\pm 1, \mathrm{Fo}_k) = 0\,, \end{gathered} \tag{15}$$

where $\psi_1(x, \mathrm{Fo}_k)$ and $\psi_2(x, \mathrm{Fo}_k)$ are the continuously differentiable and bounded functions for which the interpolation Lagrange polynomials of the nth degree [13]

$$\psi_1(x, \mathrm{Fo}_k) = \frac{2}{2n+1} \sum_{i=1}^{n} \psi_1(x_i, \mathrm{Fo}_k)(1 + x_i) \left(1 + \sum_{m=1}^{n-1} (U_m(x_i) - U_{m-1}(x_i)) \frac{T_m(x) + T_{m+1}(x)}{1 + x} \right),$$

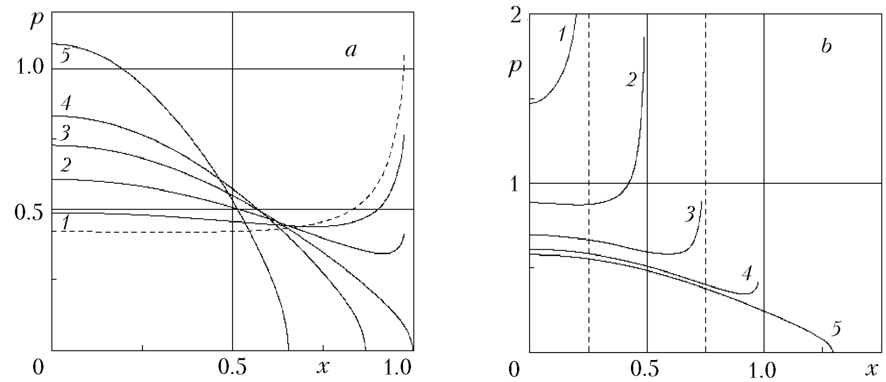


Fig. 2. Distribution of the contact pressure of the stationary problem for a die with a plane base (ν = 0.3, Bi = 2.0, P = 1, ε = 0, and c_+ = 0): a) χ = 0.5 (1), 1.0 (2), 1.298 (3), 1.5 (4), and 2.0 (5) (c_0 = 1); b) c_0 = 0.25 (1), 0.5 (2), 0.75 (3), 1.0 (4), and 1.2947 (5) (χ = 1); a) the dashed curve corresponds to the contact pressure of the force problem, b) vertical dashed curves show the asymptotes of contact pressure, which bound the size of the interaction portion.

$$\psi_2(x, \mathrm{Fo}_k) = \frac{2}{n+1} \sum_{i=1}^{n} \psi_2(x_i, \mathrm{Fo}_k)(1 - x_i^2)\left(1 + \sum_{m=1}^{n-1} U_m(x_i)\, U_m(x)\right)$$

are constructed, analogously to (14), from the polynomials

$$R_n(x) = \frac{T_n(x) + T_{n+1}(x)}{1 + x} \quad \left(x_i = \cos\left(\frac{2i - 1}{2n + 1}\pi\right), \quad i = 1, \ldots, n\right)$$

or

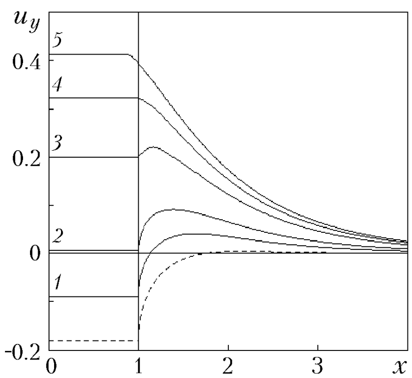
$$R_n(x) = U_n(x) \quad \left(x_i = \cos\left(\frac{i}{n + 1}\pi\right), \quad i = 1, \ldots, n\right).$$

The use of formulas (15) with interpolation Lagrange polynomials in polynomials $R_n(x)$ makes it possible to determine the actual distribution of the contact pressure for the found values of $\delta_*(\mathrm{Fo}_k)$, $\alpha_*(\mathrm{Fo}_k)$, $c_0(\mathrm{Fo}_k)$, and $c_+(\mathrm{Fo}_k)$ once the procedure described above has been used. The subdivision time step Fo_1 = 0.05 and the degree of the interpolation Lagrange polynomials n = 21 will suffice for calculations. Then the relative computational error is no higher than 5%.

Analysis of the Results. Investigation of the stationary thermoelastic contact of the die with a plane base ($f_*(x)$ = 0 and ε = 0) enables us to state that:

(a) if χ = 0 (force problem), the contact pressure has a root singularity for an arbitrary value of the half-width of the die's base c_0, and the upsetting of the die is $\delta_* > 0$ for a positive value of the pressing force;

(b) in the thermoelastic problem, an increase in the heat-generation intensity χ for a fixed value of c_0 produces a decrease in the upsetting; there comes a time where the equilibrium of the die occurs for negative values of δ_*. The contact pressure preserves its root singularity (with a coefficient of the singularity smaller than that in elastic interaction) only on condition that $c_0 < c_{\mathrm{ef}}$. When $c_0 \geq c_{\mathrm{ef}}$ the die contacts the layer along the segment $[-c_{\mathrm{ef}}, c_{\mathrm{ef}}]$, i.e., the separation of the die from the base is observed at the edge of the contact interval. The increase in χ is responsible for the decrease in c_{ef}, and, when the value of χ is fixed, a growth in the pressing force produces an increase in $|\delta_*|$ without influencing the value of the boundaries of the contact interval. These conclusions are illustrated in Figs. 2 and 3, which give the distribution of the contact pressure and the displacements of the contact-layer surface.



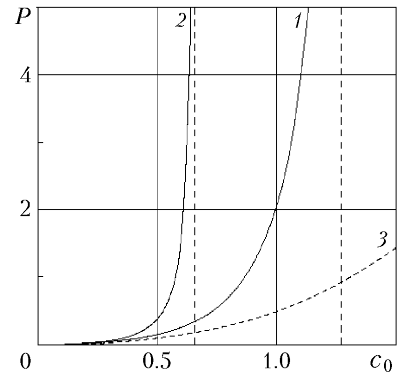


Fig. 3. Curves of displacement of the contact layer surface for $c_0 = 1$, $c_+ = 0$, and $\chi = 0.25$ (1), 0.5 (2), 1.0 (3), 1.298 (4), and 1.5 (5); dashed curve, displacement of the surface $y = 0$ in the force problem.

Fig. 4. Pressing force P vs. half-width of the contact portion c_0: $\chi = 1.0$ (1) and 2.0 (2); vertical dashed lines, asymptotes; dashed curve 3 prescribes the dependence for $P = P(c_0)$ of the elastic problem.

Further numerical investigations have shown that the quantity c_{ef} coincides with the critical half-width of the contact portion c_{cr} [7], which can be obtained by pressing a die with a parabolic base (variable contact) by an infinite force. In particular, unlike the force problem where a maximum possible value of the contact portion can be obtained in forcing-in a parabolic die, in the case of a thermoelastic problem we have such a value of $c_0(\chi)$ for each χ value that the straight line $c_0 = c_{cr}$ is a vertical asymptote for the plot of the function $P = P_0\,(c_0, \chi)$ (Fig. 4).

The conditions of change of the sign of the upsetting δ_* is insufficient for the die with a plane base to separate from the layer surface. If the half-width of the die's base c_0 is smaller than the critical value, the die interacts with the layer throughout its base, which points to the insufficient level of heat generation whose value is dependent on both the value of the parameter χ and the size of the contact portion c_0. The condition $\delta_* > 0$ points to the fact that, for these values of χ and c_0, force deformations dominate over thermal ones, whereas the opposite condition demonstrates the predominance of thermal deformations. The swelling of the contact layer surface due to the force and thermal factors is local in character: we may disregard it even approximately at a distance of $8c_0$.

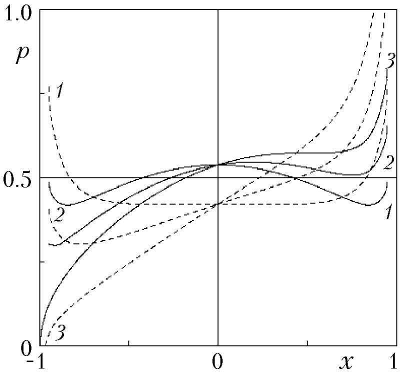
The condition of symmetry of the kernel of the integral equation and the mechanism of separation of a body on the critical value of the half-width do not allow the separation of a tilted die with a plane base from the layer surface. This conclusion is explained as follows: if the die has separated without an eccentricity, the separation occurs for critical values. It is impossible to calculate the problem for the tilted die in this case, since extension of the contact zone beyond the critical value is required. Therefore, it is possible to investigate the contact interaction of the tilted die with a plane base of length $2c_0$ only for $c_0 + \varepsilon \le c_{cr}$ for the given value of the parameter χ. As a consequence, the contact pressure will have a singularity at both ends

$$p\,(x, \mathrm{Fo}) = \frac{\psi\,(x, \mathrm{Fo})}{\sqrt{1 - x^2}} \quad \text{at} \quad c_0 + \varepsilon < c_{cr}\,,$$

or just at one end

$$p\,(x, \mathrm{Fo}) = \psi_1\,(x, \mathrm{Fo}) \sqrt{\frac{1 + x}{1 - x}} \quad \text{at} \quad c_0 + \varepsilon = c_{cr}\,.$$

Numerical calculations show that the boundary eccentricity of application of the force P to a die of length $2c_0$, which causes one edge of the die to detach itself from the layer surface, is smaller than $0.5c_0$ in the case of the elastic problem. An increase in the curvature of the die base or in the intensity of heat generation χ leads to a decrease in this boundary value of ε (Figs. 5 and 6).



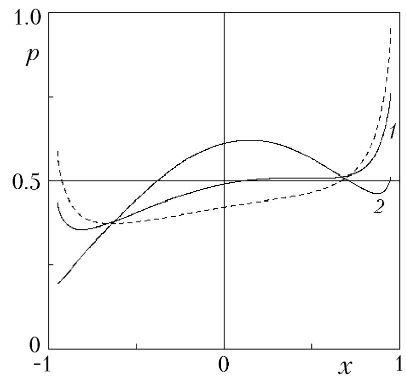


Fig. 5. Distribution of the contact pressure of the elastic problem (ν = 0.3, P = 1, c_0 = 1, and c_+ = 0) below the die with a plane (dashed curves: ε = 0 (1), 0.2 (2), and 0.4 (3)) and parabolic ($f_*(x) = 0.1x^2$, solid curves: ε = 0 (1), 0.1 (2), and 0.2 (3)) base.

Fig. 6. Distribution of the contact pressure of the thermoelastic stationary problem under the die with a plane base (Bi = 2.0, ε = 0.1, and χ = 0.5 (1) and 1.0 (2)); the dashed curve corresponds to the value χ = 0.

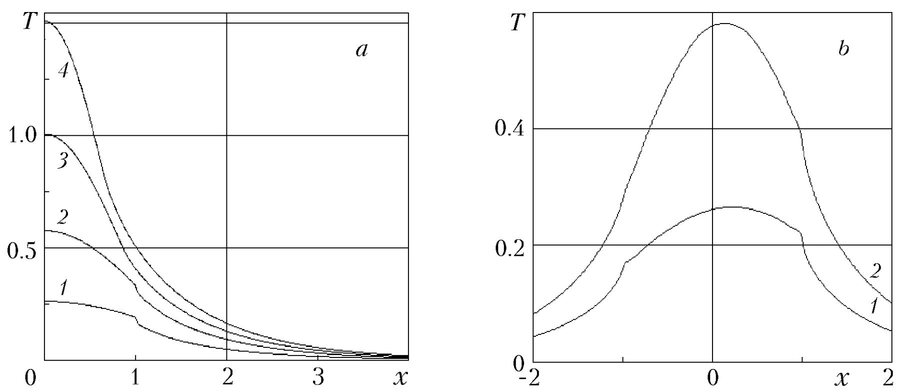


Fig. 7. Curves of variation in the stationary temperature of the surface y = 0 along the x axis in forcing-in of the die with a plane base: a) χ = 0.5 (1), 1.0 (2), 1.5 (3), and 2.0 (4) (ε = 0); b) χ = 0.5 (1) and 1.0 (2) (ε = 0.1).

The character of distribution of the stationary temperature is largely determined by the character of distribution of the contact pressure (Fig. 7). Despite the fact that the surface y = 0 beyond the interaction portion is heat-insulated, the temperature comparatively rapidly decreases. Furthermore, as the parameter Bi increases, the layer temperature decreases with thickness.

Investigations of the solution of the problem in a quasistatic formulation have shown that the value of the contact pressure monotonically becomes stationary. If the pressing force and the velocity of motion vary as

$$P_* (\mathrm{Fo}) = 1 , \quad v_* (\mathrm{Fo}) = 1 - \exp(-\mathrm{Fo}) ,$$

the duration of the transient processes for the contact pressure (6Fo) is longer than the time of the velocity reaching the stationary value (approximately 4.5Fo) (Fig. 8).

The temperature on the surface of interaction of the bodies (its level lines are given in Fig. 9) reaches the stationary value (Fo $\approx$ 7.5) somewhat more slowly, and the duration of the transient process becomes longer with distance from the surface y = 0, i.e., regions located closer to the heat-generation plane are heated somewhat more rapidly. Furthermore, the asymmetric distribution of the contact stress together with the asymmetry of the process of heat generation is responsible for the asymmetry of the temperature distribution throughout the layer thickness.

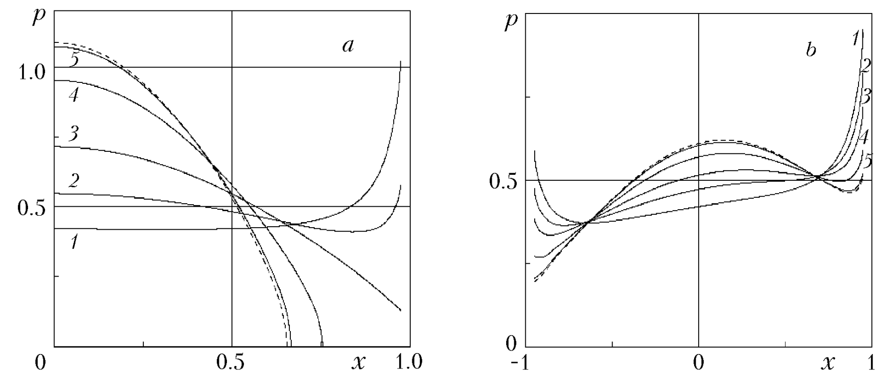


Fig. 8. Contact-pressure distribution in nonstationary heat generation for certain values of the dimensionless time Fo [1) 0, 2) 0.5, 3) 1.0, 4) 2.0, and 5) 4.0]: a) χ = 2.0 and ε = 0, b) χ = 1.0 and ε = 0.1; the dashed curves correspond to the pressure of the stationary problem.

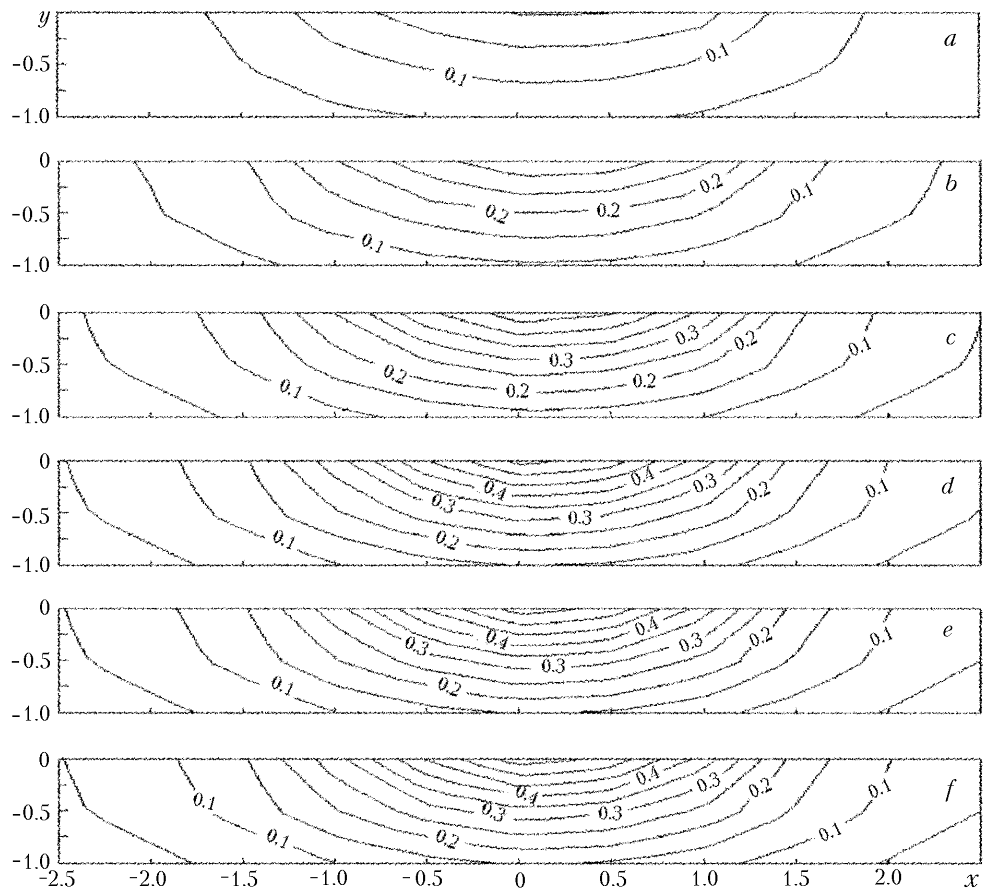


Fig. 9. Layer-temperature-level lines for the conditions of Fig. 8, b: a–f correspond to the following values of Fo: 0.5, 1.0, 2.0, 4.0, 6.0, and ∞ (stationary solution).

Thus, an analysis of the results obtained has shown that the basic characteristic of the behavior of the solution of problems of this type is the parameter χ determining the intensity of heat generation. Based on what has been said above, we may state:

1. If there is no stationary value of the pressing force or the velocity of displacement, contact stresses unboundedly increase with time.

2. If the pressing force and the velocity of displacement reach their stationary values, the contact pressure, together with the stresses, displacement, and temperature, tends to the corresponding stationary values. If the value of the half-width of the die with a plane base is lower than the critical value for a given χ, the contact stresses preserve their singularity at the end points of the contact region. Otherwise, contact-pressure regularization due to the swelling of the contact-layer surface is observed. Furthermore, an increase in the heat generation produces a monotone decrease in the half-width of the contact region to the critical value.

3. The existence of the critical value of the half-width of the contact region makes the separation of a tilted die from the layer surface impossible. Therefore, contact stresses will have a singularity either at both ends of the contact interval or just at the end where the pressing force is applied with an eccentricity.

NOTATION

a and b, boundaries of the contact region at the initial instant of time, m; Bi, Biot number; $c(\tau)$ and $d(\tau)$, time-varying boundaries of the interaction portion, m; c_{ef}, effective value of the contact interval; c_{cr}, critical value of the contact interval; $c_0(\mathrm{Fo})$ and $c_+(\mathrm{Fo})$, dimensionless boundaries of the interaction portion; E, Young modulus, N/m^2; erfc(z), error function; f, coefficient of friction; Fo, Fourier number; Fo_k, discrete value of time in numerical solution of the integral equation; Fo_*, boundary of the time interval of investigation of the behavior of the tribosystem; $G_m'(x, \mathrm{Fo}_{i,j})$, values of the integral at the instant of time Fo_i, when the boundary of the contact region has been determined at the instant Fo_m, and the integral is calculated for the instant of time Fo_j; h, layer thickness, m; $H(x, \mathrm{Fo})$, kernel of the integral equation; $H(\xi, \mathrm{Fo})$, Fourier transform of the kernel of the integral equation; $H_{\mathrm{st}}(\xi)$, Fourier transform (corresponding to the stationary solution of the problem) of the kernel of the integral equation; k, thermal diffusivity, m/sec^2; $P(\tau)$, pressing force, N; P_0, intensity of the pressing force, N; $P_*(\mathrm{Fo})$, dimensionless function of the pressing force; $p(x, \tau)$, contact pressure, N/m^2; $p(x, \mathrm{Fo})$, dimensionless function of contact pressure; $R'(x, \mathrm{Fo}_k)$, value of the sums of the right-hand side of the integral equation at the instant of time Fo_k; $R_n(x)$, polynomial of nth order, which is a combination of Chebyshev polynomials; t, integration variable; T, temperature, K; $T_n(x)$, Chebyshev polynomial of the first kind and nth order; $U_n(x)$, Chebyshev polynomial of the second kind and nth order; u_x and u_y, components of the displacement vector; $v(\tau)$, velocity of motion, m/sec^2; v_0, scale of variation in the velocity, m/sec^2; $v_*(\mathrm{Fo})$, dimensionless velocity function; x, y, z, Cartesian coordinates; x_i, zeros of Chebyshev polynomials; $y = f_0(x)$, shape function of the die's base in the contact region, m; $y = f_*(x)$, dimensionless shape function of the die's base; α, coefficient of linear thermal expansion, K^{-1}; $\alpha_0(\tau)$, angle of rotation of the die relative to the z axis; $\alpha_*(\mathrm{Fo})$, dimensionless function of the angle of rotation; γ, coefficient of heat exchange between the lower plane of the layer and the ambient medium, m^{-1}; $\Delta(x)$, kernel of the integral equation for determination of contact pressure; $\Delta(\xi)$, Fourier transform of the kernel of the integral equation; $\delta_0(\tau)$, upsetting of the die, m; $\delta_*(\mathrm{Fo})$, dimensionless function of upsetting of the die; ε, eccentricity of application of the pressing force; ε_0, computational error; η, integration variable; λ, thermal conductivity, W/(m·K); μ, positive roots of the transcendental equation of the Sturm–Liouville problem; ν, Poisson coefficient; ξ, parameter of the integral Fourier transformation; σ_x, σ_y, and σ_{xy}, components of the stress tensor, N/m^2; τ, time, sec; $\Phi(x, y, \mathrm{Fo})$, kernel of the integral transform of temperature; $\Phi(\xi, y, \mathrm{Fo})$, Fourier transform of the kernel of the integral transform of temperature; $\Phi_{\mathrm{st}}(\xi, y)$, components (corresponding to the stationary solution of the problem) of the Fourier transform of the kernel of the integral transform of temperature; χ, parameter determining the intensity of heat generation; $\psi(x, \mathrm{Fo}_k)$, $\psi_1(x, \mathrm{Fo}_k)$, and $\psi_2(x, \mathrm{Fo}_k)$, interpolation Lagrange polynomials in the expressions for contact pressure; Ω_0, initial-contact region, m. Subscripts: ef, effective value of the contact portion; cr, critical value of the contact portion; i and j, summation indices; k, discrete value of Fo; m, index of eigenvalues of the Sturm–Liouville problem; st, stationary solution of the problem; x, y, and z, components of the displacement vector and the stress tensor in the direction of the corresponding Cartesian coordinates; 0, dimensional parameters of the problem.

REFERENCES

1. S. M. Belotserkovskii and I. K. Lifanov, *Numerical Methods in Singular Integral Equations* [in Russian], Nauka, Moscow (1985).

2. I. K. Lifanov and A. V. Saakyan, Method of numerical solution of the problem on identation of a moving die into an elastic half-plane with allowance for heat release, *Prikl. Mat. Mekh.*, **46**, Issue 3, 494–501 (1982).
3. A. A. Evtushenko and O. M. Ukhanskaya, Plane contact problem of thermal elasticity in quasistationary heat release due to friction, *Izv. Ross. Akad. Nauk*, *Mekh. Tverd. Tela*, No. 5, 37–45 (1993).
4. I. I. Vorovich, D. A. Pozharskii, and M. I. Chebanov, Thermoelasticity problem for a moving die with allowance for heat release due to friction, *Prikl. Mat. Mekh.*, **58**, Issue 3, 161–166 (1994).
5. D. V. Grilitskii, A. A. Evtushenko, and V. I. Pauk, Plane contact problem of nonstationary thermoelasticity with allowance for heat generation due to friction, *Izv. Ross. Akad. Nauk*, *Mekh. Tverd. Tela*, No. 5, 62–68 (1994).
6. A. A. Evtushenko and V. I. Pauk, Nonstationary contact problem for rough bodies with allowance for heat generation due to friction, *Prikl. Mat. Mekh.*, **60**, Issue 4, 691–696 (1996).
7. A. A. Yevtushenko and V. J. Pauk, The plane transient contact problem for rough sliding bodies with wear and heat generation, *Wear*, **173**, No. 2, 39–50 (1994).
8. V. M. Aleksandrov and E. V. Kovalenko, *Problems of Continuum Mechanics with Mixed Boundary Conditions* [in Russian], Nauka, Moscow (1974).
9. I. I. Vorovich, V. M. Aleksandrov, and V. A. Babeshko, *Nonclassical Mixed Problems of Elasticity Theory* [in Russian], Nauka, Moscow (1974).
10. A. S. Galitsyn and A. N. Zhukovskii, *Integral Transformations and Special Functions in Heat-Conduction Problems* [in Russian], Naukova Dumka, Kiev (1976).
11. G. Korn and T. Korn, *Handbook of Mathematics* [Russian translation], Nauka, Moscow (1977).
12. A. I. Kalandiya, *Mathematical Methods of Two-Dimensional Elasticity* [in Russian], Nauka, Moscow (1973).
13. I. P. Natanson, *Structural Theory of Functions* [in Russian], Gostekhizdat, Moscow–Leningrad (1949).
14. M. Abramowitz and I. A. Stegun (Eds.), *Handbook of Mathematical Functions with Formulas*, *Graphs*, *and Mathematical Tables* [Russian translation], Nauka, Moscow (1979).